Quantification of Dynamic Parameters of Flexible Rotor Partially Levitated on Active Magnetic Bearing

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Quantification of Dynamic Parameters of Flexible Rotor Partially Levitated on Active Magnetic Bearing

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For every new movement,human being need to learn through either by observation or by teachings and for increasing the capacity of learning and gaining the knowledge, presence of GURU or Mentor is very necessary.

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Pramod Anandrao Shevale

(214ID1277)

Declaration of Originality

I, Pramod Anandrao Shevale, Roll Number 214ID1277 hereby declare that this thesis entitled **"Quantification of Dynamic Parameters of Flexible Rotor Partially Levitated on Active Magnetic Bearing"** represents my original work carried out as postgraduate student of NIT Rourkela, to the best of my knowledge, it contains no material previously published or written by another person, nor any material presented for the award of any other degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in thethesis. Works of other authors cited in this thesis have been duly acknowledged under the section "Bibliography". I have also submitted my original research records to the scrutiny committee for evaluation of my thesis.

I am fully aware that in thecase of any non-compliance detected in future, the Senate of NIT Rourkela may withdraw the degree awarded to me on the basis of the present thesis.

May, 2016 Pramod Anandrao Shevale

Nit Rourkela (214ID1277)

Dedicated to,

My Beloved Parents and Wife

Abstract

An unavoidable fault that generally present in the rotor system is residual unbalance, and plenty of research has been carried out to develop off-line balancing techniques in the past. But, as the present industrial need is moving towards a very high speed (supercritical) and light weight drivelines, it is important to use on-line balancing techniques instead of off-line balancing. The online balancing techniques are advantageous over offline balancing in many folds because it does not require switching off the machinery; it increases efficiency and working time of machinery. Therefore, the present work is focused on theon-linebalancing of a rotor system by estimating the distributed unbalance parameters with the help of active magnetic bearings (AMBs).

The flexible rotor model having helicallydistributed unbalances along its length and supported by two conventional bearing at both ends is well thought-out here for the analysis. Two active magnetic bearings are considered to generate controlling force for suppressing the unbalance. Then, an equivalent discretized modelis formulated by utilizing seven numberof mass-less thin discs having an eccentricity equivalent to distributed unbalance in theflexible rotor.The finite element approachhas been used to obtain a unified model.

In the present work, an identification algorithm has been developed to estimate unbalance parameters in addition with dynamic parameters of bearings i.e. conventional as well as AMBs. Equation of motion of the system hasbeen derived by considering only linear DOFs at bearing locations as generalized coordinates. Displacement at various nodes as well as current signals at AMB locations are calculated by using MATLAB Simulink. Since the present proposed algorithm requires frequency domain data to estimate unknown parameters, hence the time domain signal obtained from Simulink has been transformed into frequency domain by using Fast Fourier Transformation (FFT). Controlling force generated by AMBs is used to suppress the unbalance in the rotor. The proposed algorithm is developed on the premise of the least-squares fit method in the frequency domain. Fourth order Runge-Kutta solver is used during numerical simulation. Gyroscopic effect has been neglected in the present work.

Keywords: Active Magnetic Bearing, Flexible Rotor, Distributed Unbalance, PID Controller

Nomenclature

x_j and y_j	Translational displacements at j^{th} node of the shaft
φ_{y_j} and φ_{x_j}	Rotational displacements at j^{th} node of the shaft
$[M]^e$ and $[M]^G$	Elemental and global mass matrix
$[M_t]^{(e)}$ and $[M_r]^{(e)}$	Elemental Translational and Rotational mass matrix
$[K]^e$ and $[K]^G$	Elemental and global stiffness matrix
$[C]^e$ and $[C]^G$	Elemental and global damping matrix
$\{\eta\}^{ne}$ and $\{\eta\}$	Nodal Elemental and global displacements vector
$\{\eta_{AMB}\}$	Displacement vector of the AMB
$f_{unb}{}^{ne}$ and $f_{unb}{}^{nG}$	Nodal Elemental and global force vector
f^c	Controlling force vector of the AMB
a_0 and a_1	Rayleigh damping factors
ζ	Damping ratio
ω_{nf}	Natural frequency of the shaft
K_d	Displacement stiffness matrix of the AMB
K_i	Current stiffness matrix of the AMB

I_c	Controlling current vector of the AMB
K_P , K_D and K_I	Controlling Parameters i.e. Proportional, Derivative and Integral gain
l	Length of the element, m
L	Total length of the shaft, m
z	Z-axis coordinates of the nodes of the element
η_m and η_s	Master and slave DOFs
M_T **,** C_T and K_T	Transformed mass, damping and stiffness matrix according to master and slave DOFs
M^d , C^d and K^d	Dynamic condensed mass, damping and stiffness matrix respectively
I	Identity Matrix
T^d	Dynamic matrix
$A_1(\omega)$	Regression matrix in complex form
X_1	Vector of unknown parameters
$B_1(\omega)$	Vector of known parameters in Complex form

Subscripts

i	Current
x, y	Horizontal and vertical direction
c	Controlling

Superscripts

r and i	Real and Imaginary parts
T	Transpose of matrix
d	Dynamic condensed matrix
b	Bearing
ne	Elemental nodal
G	Global

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Chapter 1

Introduction and Literature Survey

1.1 Introduction

Imbalance present in the rotor system generates unbalance force that causes vibration in the rotor system and leads to premature failure of critical components of the machinery. If these forces are large, it may cause the failure of the whole rotor system. Thus, it is necessary to remove the imbalance of a rotor for its smooth running. At higher speeds, the imbalance in the rotor tends to bend the rotor under the influence of higher magnitude of unbalance force. Unbalance developed in rotor increases with square of spin speed of rotor. Therefore in high-speed application such as turbines, generators, compressors, pumps, motors, engines, etc. In past decades there were various balancing techniques developed that could be classified as explained in Figure 1.1.

As the present scenario is moving towards active vibration control with the help of active magnetic bearings due to the advancement in control strategy, it is time to re-think about the balancing techniques of rotors. Conventional balancing technique are very rigorous and requires machinery to be stopped that leads to ideal seating of manpower and causes heavy financial loss to industries to avoid this on-line balancing of rotating machinery is highly essential. In the present work, on-line balancing of flexible shaft having continuous imbalance along the length has been attempted. Apart from the imbalance parameters (magnitude and phase) dynamic parameters of the bearings have also been estimated.

The present work suggests a naval approach of balancing for distributed imbalance present in flexible rotor.

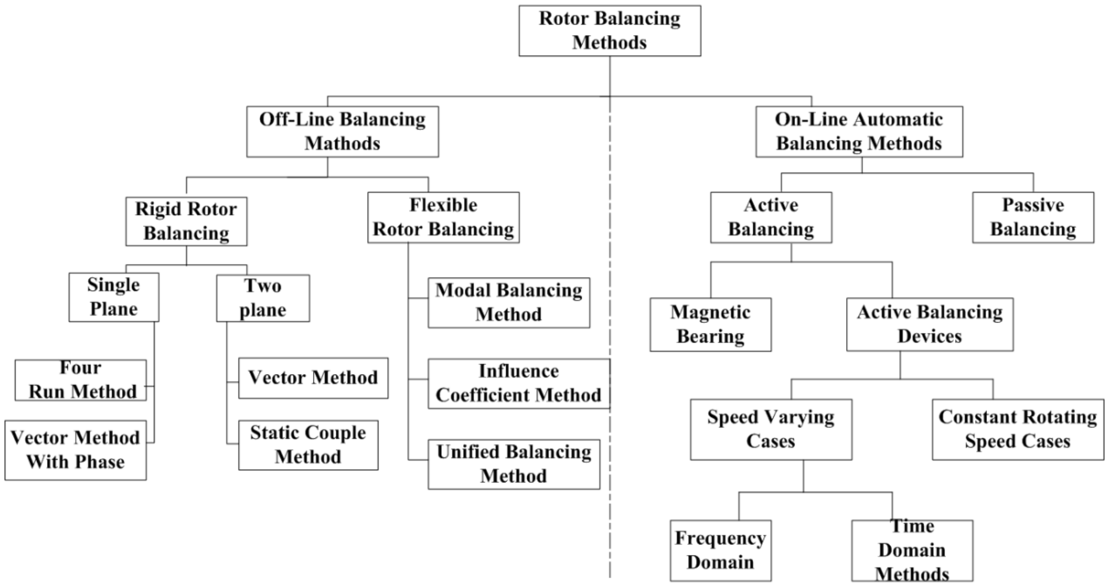

Figure 1.1 Rotor Balancing Methods

1.2 Active Magnetic Bearing

Active magnetic bearings (AMBs) works on the principle that it provides contactless motion between rotor and its housing, so there does not exist any metal to metal contact so there will be no wear and failure of magnetic bearing. Because of no wear, lubrication and sealing system also not required and it provides clean surroundings. Similarly there are lots of advantages of AMB like less maintenance cost, great rotational speeds, high stiffness, more choice in the design of bearing dynamics, and potential for vibration control. Therefore, use of AMB for on-line balancing is one of the most advanced research field.

Active magnetic bearings could be classified as Figure 1.2

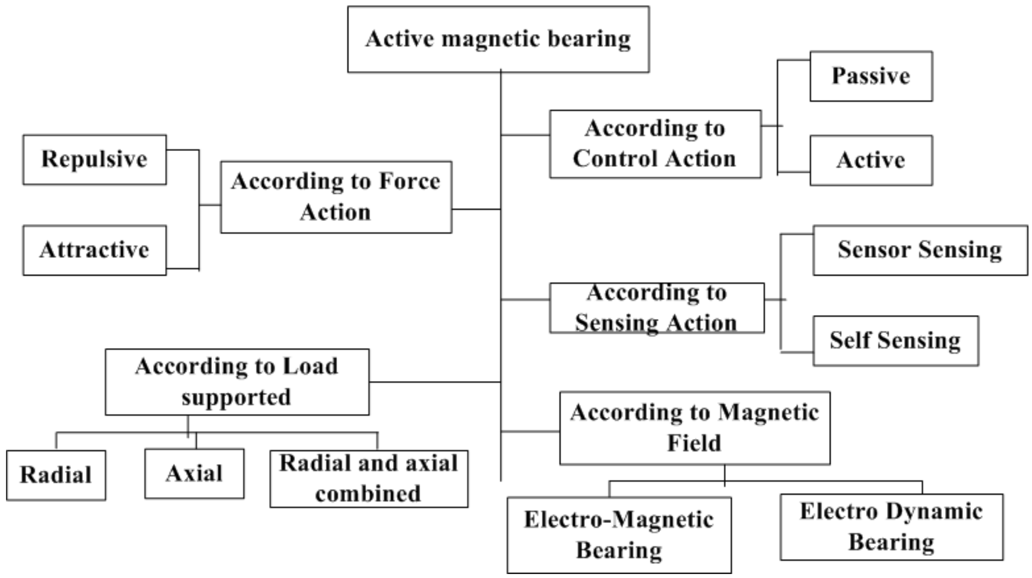

Figure 1.2 Classification of AMB

Study of AMB requires the knowledge of both mechanical and electronics engineering i.e. it is one of the typical mechatronics products. They are composed of mechanical segments consolidated with electronic components for example, sensors, power amplifiers and controllers that may be in the form of microprocessor. Application of AMB is increasing very rapidly because of drastic cost reduction and quick advance in the electronics field.

1.2.1 Working Principle of AMB

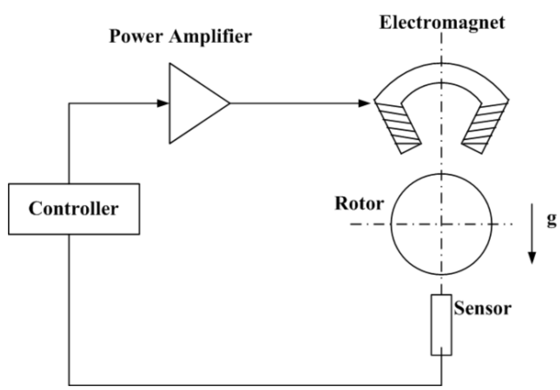

Figure 1.3 Working principle of AMB

Working principle and basic components of the AMB is shown in

Figure 1.3. A shaft is levitated with the help of electromagnet. With a specific end goal to get an active control of the shaft suspended in air, its position is measured with the help of position sensor. This position signal is then supplied to the controller, which generates the control signal corresponding to input signal. This computed signal is then enhanced by the power amplifier, to get the output as control current. At long last this control current drives the electromagnet, which creates attractive powers required for settling the rotor position and framework can be balanced out.

1.2.2 Classifications of Controllers

There are various types of controller used worldwide in the industrial application for controlling action. Some of these are explained here

I. P-Controller

Generally, P-controlleris utilized for the process of first order with storage of signal energy for stabilizing the unstable process. The fundamental use of the P controller is to diminish the steady-state error of the system because it is inversely proportional to the proportional gain factor K. therefore as value K increase error starts diminishing. Regardless of reduction, it can never figure out how to eliminate the error of the system permanently.

II. P-I Controller

P-I controller is principally used to remove steady-state error which is resulting because of P-Controller. But this controller has a negative impact in case of the speed of response and general steadiness of the system. So it is mainly used in the system where speed is not an important factor. As a result of this controller can't forecast the future blunders of the systems, it can't diminish the oscillations and rise time.

III. P-D Controller

The main aim of utilizing P-D controller is to overcome the disadvantage of the previous one by increasing the stability by enhancing control because it has the capacity to forecast error possibilities in future of the system response. Here rather than error signal derivative is considered from output response to maintain strategic distance in the value of error signal.Therefore, it is designed to change the output variable in a systematic manner and to avoid sudden changes happening in current output, which results in asudden change in the error signal.

IV. P-I-D Controller

P-I-D controller i.e. proportional-integral-derivative controller is one of the most important type used to overcome the drawbacks of other controllers. It has the excellent control dynamics which include a very fast response (i.e. short rise time), zero steady state error, no oscillation with great stability. The need of utilizing an extra device derivative gain in P-I controller is to remove the overshoot as well as oscillations produced in output response. In addition, it can be used for higher order process.

When position signal, i.e. measured displacement of a levitated object is given as input to the PID controller, it will give controlling current as an output to the coils of the electromagnet. The expression which is used for current output of controller is given as

$$i(t) = K_p x(t) + K_I \int_0^t x(t)dt + K_D \frac{dx}{dt} \quad 1.1$$

Transfer function of PID

Block diagram of PID controller is shown in Figure 1.4

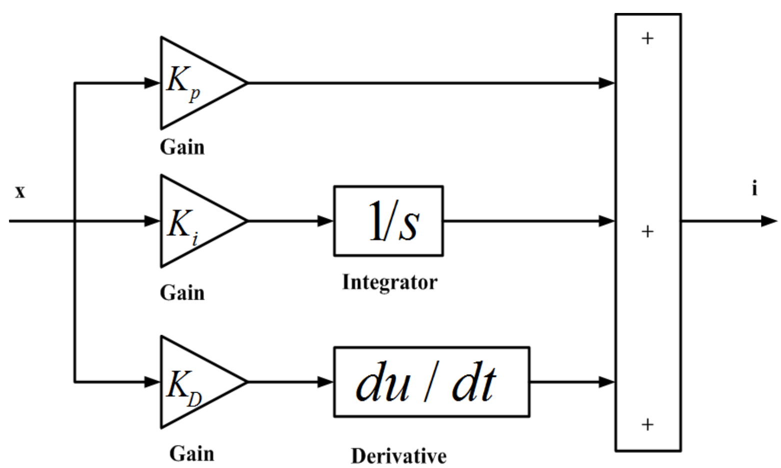

Figure 1.4 Block Diagram of PID Controller

After taking Laplace transform of Eq. (1.2) on both the sides,

$$L\left[i(t)\right] = K_p L\left[x(t)\right] + K_I L\left[\int_0^t x(t)dt\right] + K_D L\left[\frac{dx}{dt}\right] \qquad 1.2$$

Laplace transform of $\int_0^t x(t)dt$, $\frac{dx}{dt}$ and $x(t)$ are given by

$$L\left[\int_0^t x(t)dt\right] = \frac{1}{s}x(s),\ L\left[\frac{dx(t)}{dt}\right] = sx(s) - x(0),\ L\left[x(t)\right] = x(s) \qquad 1.3$$

Therefore, by substituting these values in Eq.(1.2)We get

$$i(s) = K_p x(s) + \frac{K_I}{s}x(s) + K_D s x(s) \qquad \text{or} \qquad G_c(s) = \frac{i(s)}{x(s)} = \frac{K_p s + K_I + K_D s^2}{s}$$

Where $G_c(s)$ is a transfer function PID

1.3 Background and Literature Survey

1.3.1 Background

The utilization of forces due to magnetic effect for levitating an object in the air without any contact is an old thought. Earnshaw(1848) demonstrated that passive magnetic bearing is unreliable. Because of natural propensity of the stator to attract the levitated object until they come into contact, this unsteadiness issue occurred in case of passive magnetic bearing. And because of this issue research work is prompted towards active systems. The expression 'active' implies that steady control activity should be incorporated into the framework for maintaining rotor position by adjusting the electromagnetic forces. In rotor dynamics, rotors are categorized into two groups i.e. rigid as well as flexible. We can say that every rotor is flexible, because rotor is flexible or inflexible referred according to whether first critical speed happens insides or outside the operational range of speed. The presence of continuous unbalances in flexible rotor changes the dynamic characteristics of the system and practically it is difficult to differentiate the characteristics of well-conditioned rotor and rotor with continues unbalanced while in actual working condition.

1.3.2 Importance of the study

As the present industrial need is moving towards high speed, light weight super-critical drivelines, the use of Active Magnetic Bearings (AMBs) plays a vital role in the overall functionality of any rotating machinery. AMBs supports load by using magnetic levitation to reduce friction and eliminate the lubrication system. With state-of-the-art various off-line balancing techniques have been presented, that needs rotating machinery to stop. That causes substantial financial loss to the industries. To avoid this loss limitation, in the present work we are concentrating on the on-line balancing of the rotor with the help of AMB. Although AMB is used for a purpose of on-line balancing, it gives a wide range of benefits in various fields like in industrial, medical as well as a scientific application.

1) It minimizes friction as well as totally eliminates wear which is intrinsic in the conventional bearing by avoiding physical contact between rotor and bearing.
2) Magnetic bearing by utilizing less power operates at very high-speed application like in spindle.
3) It offers diagnostic capabilities as well as advanced monitoring. This considers early identification of emerging faults. Also, optimize the performance of the system.
4) It is fit for exact control of rotor position, lessening of vibration and with the use of active control it controls flexible structural resonance.
5) Because of no friction, it eliminates lubrication in the gap of rotor and bearing and provides neat and clean operating environment.
6) These are able to work under an extreme situation such as low or high pressures and in a wide range of temperature applications.

1.3.3 Literature Survey on active magnetic bearing

Habermann, et al. (1984) discussed that AMB is able to provide an optimum damping for the flexible rotor. Here they explained all the major advantages of AMB as well as a working principle of it and extended the research in the field of AMB to empower the optimum damping of a flexible shaft. By using various types of loop such as (a) Standard control loop (b) Translational as well as tilting movement control loop (c)Automatic balancing and

Rotating coordinates control loop.(d) In flexible rotor, an exceptional control loop for damping of the shaft bending frequency, they presented damping as well as stiffness behavior of AMB.

Dussaux and Michel (1990) reviewed the working principle, fundamental information, innovations as well as typical benefits of AMB. Then he focused on some worldwide fields where AMB is used like aerospace applications, machine tool applications, light industry applications as well as heavy industry applications.

Bleuler (1992) said that there are two main application fields of magnetic levitation i.e. in rotor bearing and in transportation, and they proposed the well-ordered classification of principles of basic levitation. A new active bearing which is self- sensing, i.e. without using a gap sensor is presented here.

Vischer and Bleuler (1993) presented a theoretical and experimental approach to develop an active magnetic bearing with no use of sensors, i.e. there will be no sensing hardware to sense position signal. Here a position sensor is supplanted with the use of simple device for measuring current in amplifier so it is called self-sensing AMB. They developed number of self-sensing AMB, test them experimentally, and concluded that for controlling system, current estimation in power amplifier is the only signal required.

Antila, et al. (1998) predicted the performance of radial AMB by utilizing non-linear two dimensional FEM (finite element method). They came to conclusion that the performance of radial AMB does not satisfactorily predicted by the linear magnetic circuit model. In case of non-linear region, FEM model of radial AMB is correct enough. They found 7-8% error in examining the forces in non-linear region and that can be enhanced with more precise material information.

Stumberger, et al. (2000) exhibited an optimization of radial AMB. By utilizing differential evolution-a stochastic direct search algorithm, radial bearing is numerically optimized. By utilizing 2D FEM non-linear solution of the magnetic vector potential is resolved. Maxwell’s stress tensor method is used for calculating force. At last they compared parameters of optimized as well as non-optimized bearing.

Ji, *et* al. (2000) investigated non-linear behavior of inflexible (rigid) shaft which is suspended by using AMB. Normal-form technique is utilized to examine horizontal as well as vertical vibrations on the center manifold close to the double-zero degenerate point. Because of the effect of weight of rotor, different normal forms are obtained in horizontal as well as vertical directions. They also discussed the autonomous and non-autonomous cases for vertical vibrations.

Ji andHansen (2000) by assuming primary as well as internal resonance investigation has been done for non-linear response of rotor levitated by AMB. Four first order differential equation has been acquired by utilizing technique of multiple scales which describes phases of vibrations, modulation of amplitudes in vertical and horizontal directions. It is numerically determined solidness of solution as well as steady state response from lessened system.

Yang and Lin (2002) explained that discrete unbalance balancing techniques couldn't be used for balancing of rotor with distributed unbalance. So they extend their approach towards balancing of continues unbalances of the rotor. Here it is considered that shaft eccentricity is distributed in piecewise polynomials. For distributed unbalance of shaft, they provided a finite element approach as well as an identification algorithm. And they concluded that this proposed algorithm is effective and acceptable in case of non-isotropic rotor bearing system and great damping.

Tiwari (2004) developed a general algorithm for estimating various parameters like stiffness, damping, residual unbalances etc. of four-DOFs rotor-bearing-system. Similarly, he developed a special algorithm for identifying residual unbalance and dynamic parameters of bearing. By measuring the responses by rotating the rotor in clockwise and anti-clockwise alternatively gives a good conditioned algorithm.

Mani, et al. (2006) studied the rotor dynamic bearing system having breathing cracks in it for developing an algorithm for detection of damage. They assumed that, rotor is supported by conventional bearing as well as an AMB is utilized for applying controlling force at the mid-shaft. Multiple scale analysis is used for identifying combination resonance between operating speed, critical frequency of the shaft and the excitation frequency of AMB. Here because of excitation frequency can be picked subjectively, this condition of resonance can be fulfilled for any blend of critical and operating speed. They concluded that the amplitude of the component, which is in vibration response at critical frequency, is proportional to the magnitude of magnitude of stiffness, which is time dependent, presented by crack.

Knospe (2006) exhibited experimental results for two test apparatus, which shows the capability of AMB for the active concealment of machining prattle. In first apparatus, he considered prototype of magnetic bearing shaft at 32000RPM, which is constructed and modeled in participation with Cincinnati Milacron, and in second apparatus, he considered turning operation for active control of chatter presents in it.

Li, et al. (2006) developed an exact nominal model for vigorous controller combination and examination. They employed a blend of analytical modeling, system upgrading as well as identification of each part of system and again for whole system. An orderly method was created to develop the system at part level. A precise rotor model was gotten through the expository demonstrating and model overhauling and parameters of AMB recognized straightforwardly.

A methodology for modeling and controlling of a flexible rotary shaft, which is levitated by magnetic forces through active magnetic bearing, is developed by Arredondo, et al.(2007). Here three important dynamic effects considered, i.e. (a) Rigid shaft dynamics; (b) The flexible dynamics; (c) The unbalanced motion dynamics, which are present in the system, and super positioned to develop modeling. Newton’s law is applied to calculate rigid part of model and then by utilizing finite element analysis, flexible dynamics has been calculated and at last modeling of unbalance motion dynamics have been done. This methodology has been used for getting linear model as well as non-linear model of AMB.

Lei and Palazzolo (2008) presented a methodology for examination and design of system with magnetic suspension which involves a flexible rotor model. A finite element model of flexible rotor is gathered and after that utilized to build-up a magnetic suspension compensator which will give great stability and disturbance rejection. They formulated a flexible shaft rotor model for high speed flywheel supported with magnetic bearing and simulated it to avoid disturbance.

Tiwari and Chakravarthy (2008) described two separate algorithms for estimating dynamic parameters of bearing and residual unbalances for framework of rigid rotor-bearing. Both of these algorithms are demonstrated with the numerical simulation and experimental measurement. In case of the first method, impulse method is used for measurement of response in both directions, i.e. horizontal and vertical direction and in second case, responses resulting from three different configurations of unbalances are used. These algorithms are tested numerically as well as experimentally with noise and without noise and obtained excellent results.

Lal and Tiwari (2013) developed an algorithm for estimating the dynamic parameters of bearing as well as coupling in the system of turbo-generator. And here they considered the effect of residual unbalances along the predefined planes, forces due to misalignment and moment at coupling. Some non-measurable degrees of freedoms, i.e. rotational DOFs are present at bearing and coupling location and that can’t be removed with dynamic condensation so the high frequency condensation technique is executed here to remove these DOFs. Then for estimating the bearing and coupling parameters which are independent of speed, this developed algorithm is illustrated in a simplified numerical cases.

Lal and Tiwari (2012) developed an algorithm for identifying the parameters of multiple faults in a turbo-generator model by using the information of forced response. They considered a simple turbo-generator model with rigid rotor and flexible coupling as well as bearing. They accounted effect of force due to misalignment created in coupling used for connecting driving and driven shaft in the system. Lagrange's eq.is used to generate an eq. of motion. To delineate the viability of developed algorithm numerical trials have been performed and it is also tested for measurement noise.

Chougale and Tiwari (2013) developed an algorithm for two plane motion rotor-bearing system which fully suspended by AMB to identify the residual unbalance as well as AMB dynamic parameters. Run-up and run-down data is used for this method to determine AMB parameters and unbalances at predefined planes of the rotor. This identification algorithm is depending on least-squares fit techniques in frequency-domain and it is simulated numerically to determine its reliability.

Singh and Tiwari (2015) investigated the behavior of framework with rotor-bearing having breathing crack in it. AMB is used to generate an equivalent controlling force to suppress the vibration produced because of crack as well as to suppress residual unbalances. Full-spectrum analysis is used for identifying the coefficient of crack force at multiple harmonics by exciting the rotor in same and reverse direction of rotor spin. By using these coefficients in this algorithm various parameters like damping, additive crack stiffness and unbalance of disc are identified.

Singh and Tiwari (2016) studied the characteristics of vibration of Jeffcott rotor supported on AMB having crack in it with an offset disc. Cracked rotor with two plane motion is modeled with consideration of gyroscopic effect because of offset disc as well as SCEF (Switching Crack Excitation Function). While developing an identification algorithm dynamic reduction is used to eliminate rotational DOFs which may cause practical difficulty in true measurement. It is tested for noise also.

1.4 Problem definition

The main purpose of the current project is to formulate a simple flexible rotor with distributed imbalances confined in it along its length. Generally, we can say that every rotor is flexible, because is it flexible or inflexible rotor is referred according to whether first critical speed happens insides or outside the operational speed-range of rotor. Presence of continuous unbalances in flexible rotor changes the dynamic characteristics of the system and practically it is difficult to differentiate the characteristics of well-conditioned rotor and rotor with continues unbalanced while in actual working condition. So in the present work we are concentrating on on-line balancing of flexible rotors. Flexible shaft with continuous unbalance supported on two conventional bearings at its end is considered and two active magnetic bearings are used to suppress this unbalanced force by generating controlling force.

1.5 Motivation of the present work and objectives

As per the introduction and literature survey discussed above, it could be concluded that in the field of rotor dynamics as the present industrial need is moving towards a very high speed, light weight super critical drivelines, it is necessary to use on-line balancing techniques rather than off-line balancing techniques. The use of Active Magnetic Bearings (AMBs) plays a vital role in the overall functionality of any rotating machinery. AMBs supports load by using magnetic levitation to reduce friction and eliminate the lubrication system. With state-of-the-art various off-line balancing techniques have been presented, that needs rotating machinery to stop. That causes substantial financial loss to the industries. To avoid this loss limitation, in the present work we are concentrating on the on-line balancing of the rotor with the help of AMB. Although AMB is used for a purpose of on-line balancing, it gives wide range of benefits in various fields like in industrial, medical as well as scientific applications.

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(4) It is fit for exact control of rotor position, lessening of vibration and with the use of active control it controls flexible structural resonance.
(5) Because of no friction, it eliminates lubrication in the gap of rotor and bearing and provides neat and clean operating environment.

Chapter 2

System Modeling

2.1 Introduction

A flexible rotor bearing system having distributed unbalance confined along its length and for suppressing it, use of AMB is considered here. Theoretical model analogous to flexible shaft is formulated and explained here. Finite element formulation for obtaining eq. of motion of the present system with the use of Rayleigh–Ritz method and without the use of gyroscopic effect is explained. Flexible rotor is modeled theoretically as shown in below figures and analyzed by using a discrete unbalance approach for on-line balancing on it. Detail description of flexible shaft with continues unbalance and its formulated discrete model is given. Present section is concerned with the development of an algorithm for estimating AMB dynamic parameters as well as conventional bearing parameters and residual unbalances.

2.2 Model description

In this article, a flexible rotor-bearing model with distributed unbalance over the length is attempted to balance with the help of active magnetic bearings (AMBs) as shown inFigure 2.1 is considered for the study.It is composed of a flexible shaft supported by two conventional bearings at both ends and two AMBs at two different locations in mid span. It is a verifiable truth that if the rotor is working at well beneath of first critical speed, then there will not be any significant bending of the shaft so it is considered as rigid, but if the operating speed is at risk to achieve first critical speed, then rotor starts to twist and this bending is significant because it will add an extra unbalance. This extra unbalance because of the twisting of the shaft is termed as distributed unbalance. It is assumed that unbalance is helically distributed along the length of a rotor. An equivalent theoretical model formulated and which is used for the analysis is shown in Figure 2.2. It contains seven no of mass-less disc having an eccentricity equivalent to distributed unbalance as in flexible rotor. It is assumed that distributed unbalance is replaced by disc so now shaft will become rigid. Discrete unbalance approach is now used to balance it and for evaluating dynamic parameters.

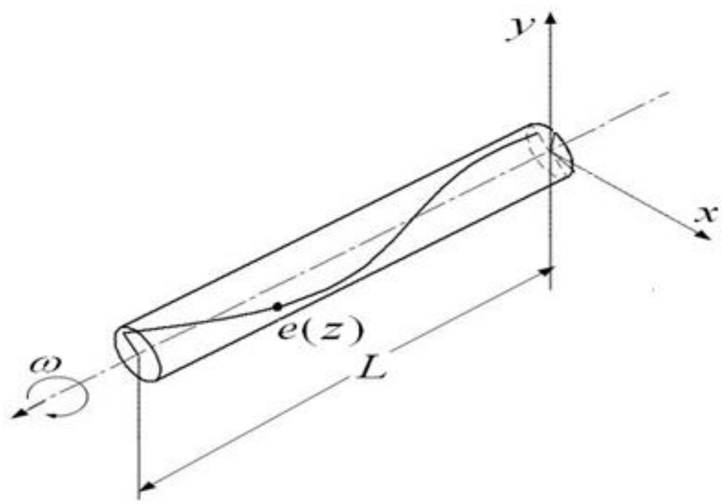


Figure 2.1 Flexible Rotor with Helical Unbalance

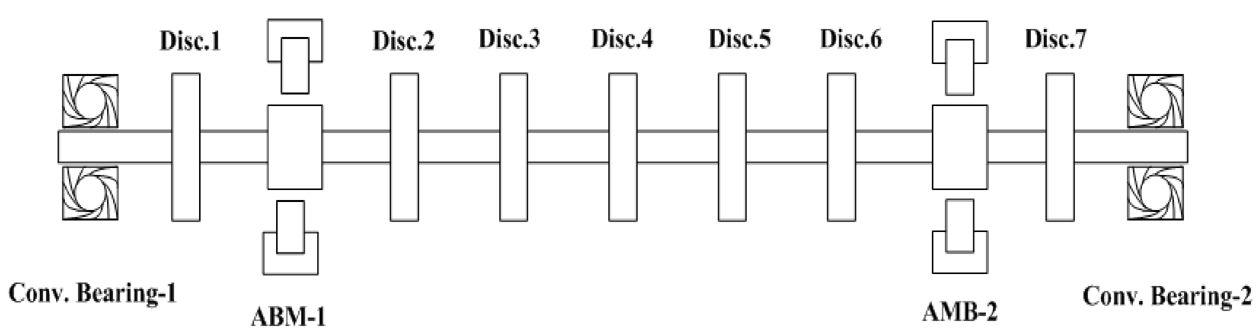


Figure 2.2 Discrete model of Flexible Rotor

The assumptions considered during modeling of rotor frameworks and other specifications of the rotor also expressed here. Eq. of motion of rotor system having AMBs, mass less rigid and very thin discs and conventional bearing has been derived which is then used to obtain forced response and to develop an identification algorithm. Two plane motion of the shaft is considered here, i.e. there will be two linear and two linear as well as two rotational degrees of freedom at each node of the shaft. Total DOFs of system will be $4(n+1)$, where n is number of elements in which system is divided.

2.3 Equation of motion

The total system is divided into number of individual sub-models like rotor-model, conventional bearing-model, AMBs-model and disc-model. Then the eq. of motion for every individual sub-model is derived and by doing assemble of all equations, final global equation of motion has been obtained.

2.3.1 Equation of motion of rotor model

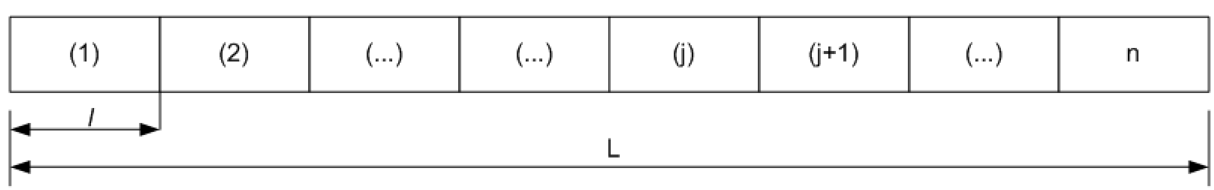


Figure 2.3 Discretization of rotor into elements

A rotor shaft is discretized into ‘n’ no of elements as shown in above Discretization of the rotor into ‘n’ no number of elements create total *‘(n+1)’* number of nodes. Then each individual element will be taken for the analysis separately and elemental eq. of motion for that model will be derived. Here in present work two translational and two rotational displacements for the shaft has been considered. Therefore, there will be total four degrees of freedom at each node. So that total degree of freedoms will be equal to *4(n+1).*

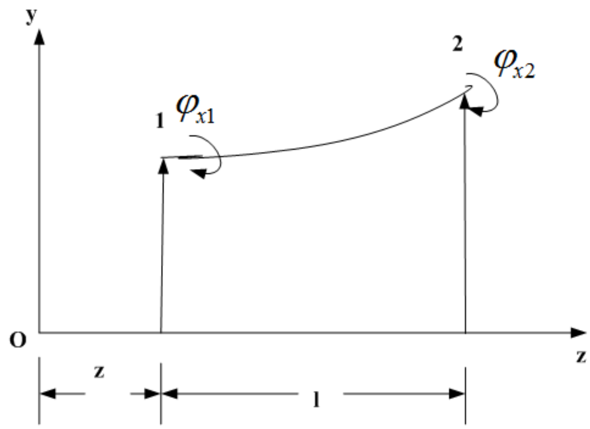


Figure 2.4 A shaft element of length *l* subjected to angular and linear displacements

Consider a shaft element is at a distance z in *y*-*z* plane as shown in Figure 2.4. It is considered that the rotor is rotating in two planes. So at each node, there will be two translational and two rotational degrees of freedom. Let nodal translational displacements of the rotor represented by x and y, and rotational displacements of shaft represented by φ_y and φ_x in *z*-*x* and *y*-*z* planes, respectively. With the use of subscripts 1 and 2, these displacements would belong to respective nodes. Here x_1 and y_1 are linear displacements at node 1 and similarly x_2 and y_2 are linear displacements at node 2, φ_{y1} and φ_{x1} are rotational displacements at node 1 and $\varphi_{y2}, \varphi_{x2}$ are rotational displacements at node 2. The length of the element is l.

It is assumed that cross-section of the shaft, dimension, material properties is assumed uniform throughout. Equation of motion for element of shaft with considering the proportional damping is shown in Eq. (2.1)

$$[M]^e \{\ddot{\eta}\}^{ne} + [C]^e \{\dot{\eta}\}^{ne} + [K]^e \{\eta\}^{ne} = \{f_{unb}\}^{ne} \tag{2.1}$$

And

$$\{\eta\}^{(ne)} = \{x \quad \varphi_y \quad y \quad \varphi_x\}^T$$

Where $[M]^{(e)}, [C]^{(e)}$ and $[K]^{(e)}$ are elemental mass, damping and stiffness matrix, $\{\eta\}^{(ne)}$ and $f_{unb}^{(ne)}$ are elemental nodal displacement and force vector.

Damping Matrix

Proportional elemental damping matrix $[C]^e$ of the rotor can be expressed in the terms of elemental mass and stiffness matrix $[M]^e$ and $[K]^e$ respectively as shown in below Eq. (2.2)

$$[C]^e = a_0 [M]^e + a_1 [K]^e \qquad 2.2$$

Where a_0 and a_1 are Rayleigh damping factors. The relation between damping ratio ζ and the natural frequency ω_{nf} in terms of Rayleigh factors a_0 and a_1 is given as (Clough and Penzien 1993).

$$\zeta_n = \frac{a_0}{2\omega_{nfn}} + \frac{a_1 \omega_{nfn}}{2} \qquad 2.3$$

And

$$\zeta_m = \frac{a_0}{2\omega_{nfm}} + \frac{a_1 \omega_{nfm}}{2} \qquad 2.4$$

Rayleigh damping factors a_0 and a_1 can be evaluated by the solution of a pair of simultaneous equations, if damping ratio ζ_n and ζ_m are associated with two specific known frequencies ω_{nfn} and ω_{nfm} respectively. The Eq. (2.3) and (2.4) can be expressed in the matrix form as follows

$$\begin{Bmatrix} \zeta_m \\ \zeta_n \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} \omega_{nfm}^{-1} & \omega_{nfm} \\ \omega_{nfn}^{-1} & \omega_{nfn} \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} \qquad 2.5$$

From the above Eq. (2.5) a_0 and a_1 can be calculated as

$$\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = 2\frac{\omega_{nfm}\omega_{nfn}}{\omega_{nfn}^{2}-\omega_{nfm}^{2}}\begin{bmatrix} \omega_{nfn} & -\omega_{nfm} \\ \omega_{nfn}^{-1} & -\omega_{nfm}^{-1} \end{bmatrix}\begin{Bmatrix} \zeta_m \\ \zeta_n \end{Bmatrix} \qquad 2.6$$

Where ω_{nfm} and ω_{nfn} are fundamental and highest natural frequencies of the system. In the present article lowest and highest natural frequencies are taken. Therefore, by using the values of a_0 and a_1 obtained from Eq. (2.5) in Eq. (2.6), damping matrix of the system is obtained.

Then final equation of motion for rotor element is

$$[M]^e\{\ddot{\eta}\}^{ne} + [C]^e\{\dot{\eta}\}^{ne} + [K]^e\{\eta\}^{ne} = f_{unb}^{\ ne} \qquad 2.7$$

2.3.2 Equation of motion of bearing model

In the present work flexible rotor having continuous unbalance has been supported on two different flexible bearings at its both ends. It is assumed that the mass of the bearing is negligible. Bearing stiffness matrix and damping matrix are calculated for the bearing model and then it is combined with rotor stiffness and damping matrix respectively during globalization.

$$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}\{\eta\}^b + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix}\{\dot{\eta}\}^b = \{0\} \qquad 2.8$$

Bearing stiffness as well as damping matrix are mentioned in detail in appendix.

2.3.3 Equation of motion of AMB model

Two AMBs are used in the system for exerting a controlling force at corresponding node of the rotor shaft to suppress the unbalances. A standard PID controller is used in the model. Controlling force generated with AMB is defined as in Eq. (2.9)

$$f^c = [K_d]\{\eta_{AMB}\} + [K_i]\{I_c\} \qquad 2.9$$

Where,

$$Kd = \begin{bmatrix} k_{sx1} & 0 & 0 & 0 \\ 0 & k_{sy1} & 0 & 0 \\ 0 & 0 & k_{sx2} & 0 \\ 0 & 0 & 0 & k_{sy2} \end{bmatrix}, K_i = \begin{bmatrix} K_{ix1} & 0 & 0 & 0 \\ 0 & K_{iy1} & 0 & 0 \\ 0 & 0 & K_{ix2} & 0 \\ 0 & 0 & 0 & K_{iy2} \end{bmatrix}$$

$\eta_{AMB} = \{x_3 \quad y_3 \quad x_9 \quad y_9\}^T$, and $I_c = \{I_{x3} \quad I_{y3} \quad I_{x9} \quad I_{y9}\}^T$

Where K_d is displacement stiffness matrix of AMB and K_i is the current stiffness matrix, η_{AMB} is displacement vector of AMB and I_c is controlling current of AMB

Mathematically, in case of PID controller I_c of AMB can get by following calculation,

$$I_c = \begin{bmatrix} K_p & 0 & K_d & 0 & K_I & 0 \\ 0 & K_p & 0 & K_d & 0 & K_I \end{bmatrix} \begin{Bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \\ \bar{x} \\ \bar{y} \end{Bmatrix}$$

2.3.4 Global Equation of motion

After getting all elemental equations of motion for all discretized parts of the system, assembling of all equation can be done. So the global equation of motion obtained after combining all is

$$[M]^G \{\ddot{\eta}\} + [C]^G \{\dot{\eta}\} + [K]^G \{\eta\} = \{f_{unb}\} - \{f^c\} \quad 2.10$$

Controlling force obtained from AMB is used to suppress the continuous unbalance in the shaft, therefore it is taken as negative at left side.

Chapter 3

Identification Algorithm by applying Dynamic Condensation Scheme

3.1 Dynamic condensation Scheme

In the present work, motion of the system is considered in two planes i.e. x-z and y-z plane. So the system has total $4(n+1)$ number of DOFs where n is the number of elements. These high DOFs will create problems in accurate estimation so it is important to wipe out unnecessary DOFs from the system.

Basically, there are two main condensation schemes are available, i.e. static and dynamic. Both the condensation schemes are utilized for eliminating unnecessary degrees of freedoms from global equation of motion. A static condensation method is developed by the Guyan (1965) and it is the easiest method which gives accurate results in a range of low frequencies. It is used in the systems in which inertial terms are mostly negligible. Then the dynamic condensation method is developed with modification in the static condensation method for obtaining exact results at any frequency. During condensation the degrees of freedom which are wiped out are called as slave DOFs and which are remains in the equation are called as masters DOFs. Generally, DOFS corresponding to bearing locations, locations of disc or balancing planes and location of any external forces are considered as master DOFs. DOFs which are non-critical like rotational DOFs as well as at intermediate locations of the rotor where no external force presents are considered as slave DOFs. In the present work dynamic condensation scheme has been employed during the development of identification algorithm to wipe out linear as well as rotational degrees of freedoms at non-bearing locations from the equation of motion, because it will create trouble in exact measurement. At the very first stage of condensation, all the matrices i.e. mass, damping stiffness and unbalanced force vector is transformed according to the sequence of masters and slave DOFs and then these are divided into sub-matrices and sub-vectors.

Transformation matrix can be written as

$$M_T = \begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix}; \qquad C_T = \begin{bmatrix} C_{mm} & C_{ms} \\ C_{sm} & C_{ss} \end{bmatrix}; \qquad K_T = \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix};$$

$$\eta_T = \begin{Bmatrix} \eta_m \\ \eta_s \end{Bmatrix}; \; f_T = \begin{Bmatrix} f_m \\ f_s \end{Bmatrix}$$

After transformation of global equation and in frequency domain it becomes

$$\left[-\omega^2 M + j\omega C + K\right]\begin{Bmatrix} \eta_m \\ \eta_s \end{Bmatrix} = \begin{Bmatrix} f_m \\ f_s \end{Bmatrix} \tag{3.1}$$

This transformed frequency domain equation becomes explode in two sub-equations in such a way that

$$-\omega^2\left[M_{mm}\right]\{\eta_m\} - \omega^2\left[M_{ms}\right]\{\eta_s\} + \left[K_{mm}\right]\{\eta_m\} + \left[K_{ms}\right]\{\eta_s\} = \{f_m\} \tag{3.2}$$

And

$$-\omega^2\left[M_{sm}\right]\{\eta_m\} - \omega^2\left[M_{ss}\right]\{\eta_s\} + \left[K_{sm}\right]\{\eta_m\} + \left[K_{ss}\right]\{\eta_s\} = \{0\} \tag{3.3}$$

Where ω the spin speed of the rotor and Eq. (3.3) is can be reorganized in the following form

$$\{\eta_s\} = -\left(\left[K_{ss}\right] - \omega^2\left[m_{ss}\right]\right)^{-1}\left(\left[K_{sm}\right] - \omega^2\left[M_{sm}\right]\right)\{\eta_m\} \tag{3.4}$$

With an identity matrix, equation can be writing like

$$\{\eta_m\} = [I]\{\eta_m\} \tag{3.5}$$

Eq. (2.18) and (2.19) can be recombined as

$$\begin{Bmatrix} \eta_m \\ \eta_s \end{Bmatrix} = \left[T^d\right]\{\eta_m\}$$

Where $\left[T^d\right]$ is transformation matrix and is obtained by

$$T^d = \begin{bmatrix} I \\ -(K_{ss} - \omega^2 M_{ss})^{-1}\left(K_{sm} - \omega^2 M_{sm}\right) \end{bmatrix} \tag{3.6}$$

Where $[I]$ is identity matrix of size of masters DOFs and

K_{ss} and M_{ss} stiffness and mass matrix corresponding to slave DOFS respectively

K_{sm} and M_{sm} stiffness and mass matrix corresponding to slave and master DOFS respectively

By using this transformation matrix, dynamic matrices of stiffness, damping and mass has been obtained respectively as

$$K^d = T_d' \times K_T \times T_d, \qquad C^d = T_d' \times C_T \times T_d$$

$$M^d = T_d' \times M_T \times T_d, \qquad \{f_{unb}\}^d = [T_d]' \times \{f_{unb} + f_c\}_T$$

K_T, C_T and M_T are new arranged matrices according to master DOFs and slave DOFs.

Condensed equation of motion

$$[M]^d\{\ddot{\eta}_m\} + [C]^d\{\dot{\eta}_m\} + [K]^d\{\eta_m\} = \{f_{unb}\}^d \tag{3.7}$$

Where $\{\eta_m\} = \{x_1 \quad y_1 \quad x_3 \quad y_3 \quad x_9 \quad y_9 \quad x_{11} \quad y_{11}\}$ is Master Displacements Vector

3.2 Identification Algorithm

Eq. (3.7) describes a governing equation of motion having only masters DOFs of flexible rotor-system. This equation is then utilized to obtain a forced response of the system. This obtained response is assumed as actual response and then utilized for developing identification algorithm for estimating the dynamic parameters of conventional bearing, active magnetic bearing, and simultaneously it is used to estimate the magnitude of residual unbalances.

In Eq. (3.7) force vector can be rewritten as $\{f_{unb}\}^d = F_{unb}e^{j\omega t}$ and this F_{unb} involve the data of residual unbalance force and AMB force. Similarly, displacement as well as current vectors can be rewritten in the form of

$\eta_m = N_m e^{j\omega t}$ and $I_c = I_m e^{j\omega t}$

Magnitude and phase information of displacement, current and unbalances is stored into N_m, I_m and F_{unb} respectively as well as these quantities are in the form of complex. Now substituting these in Eq. (3.7), equation becomes

$$\left(-\omega^2\left[M^d\right] + J\omega\left[C^d\right] + \left[K^d\right]\right)N_m = F_{unb} \tag{3.8}$$

The above equation is in the frequency domain and contains all the known and unknown parameters of the system. Now by separating dynamic damping and stiffness matrices

$$\left(-\omega^2\left[M^d\right] + J\omega\left[C^{bd} + C^{rd}\right] + \left[K^{bd} + K^{rd}\right]\right)N_m = F_{unb} \tag{3.9}$$

Where C^{rd} is the rotor dynamic damping matrix, K^{rd} is rotor dynamic stiffness matrix and these are known matrix along with the M^d also. But C^{bd} and K^{bd} are bearing dynamic damping and stiffness matrix respectively, and these contain unknown parameters. Also F_{unb} contains unknown dynamic parameters of AMB. Therefore, this Eq. (3.9) can be rearranged to separate known, as well as unknown quantities of the system. Known parameters will be at Right Hand Side (RHS) of equation and Unknown parameters will be at Left Hand Side (LHS) of equation

Therefore, equation becomes

$$\left(J\omega\left[C^{bd}\right]+\left[K^{bd}\right]\right)N_m - F_{unb} = -\left(-\omega^2\left[M^d\right]+J\omega\left[C^{rd}\right]+\left[K^{rd}\right]\right)N_m \qquad 3.10$$

LHS of the equation can be arranged in such a way that all known quantities such as obtained forced responses, model parameters will be in the regression matrix at LHS. Dynamic parameters like stiffness and damping factors of conventional bearing, dynamic parameters of AMB, and residual unbalances are in vectors at LHS.

Therefore, equation becomes

$$\left[A_1\right]\left\{K^b\right\}+\left[A_2\right]\left\{C^b\right\}+\left\{\left[A_3\right]\left\{K_d\right\}+\left[A_4\right]\left\{K_i\right\}\right\}-\left[A_5\right]\left\{U\right\}=B_1 \qquad 3.11$$

Where

$$\left\{K^b\right\}=\begin{Bmatrix} K_{xx1}\\ K_{xy1}\\ K_{yx1}\\ K_{yy1}\\ K_{xx2}\\ K_{xy2}\\ K_{yx2}\\ K_{yy2}\end{Bmatrix};\quad \left\{C^b\right\}=\begin{Bmatrix} C_{xx1}\\ C_{xy1}\\ C_{yx1}\\ C_{yy1}\\ C_{xx2}\\ C_{xy2}\\ C_{yx2}\\ C_{yy2}\end{Bmatrix};\quad \left\{K_d\right\}=\begin{Bmatrix} K_{sx1}\\ K_{sy1}\\ K_{sx2}\\ K_{sy2}\end{Bmatrix};\quad \left\{K_i\right\}=\begin{Bmatrix} K_{ix1}\\ K_{iy1}\\ K_{ix2}\\ K_{iy2}\end{Bmatrix};\quad \left\{U\right\}=\begin{Bmatrix} U_1\\ U_2\\ .\\ .\\ .\\ U_7\end{Bmatrix}$$

And

$$B_1=-\left(-\omega^2\left[M^d\right]+J\omega\left[C^{rd}\right]+\left[K^{rd}\right]\right)N_m$$

Eq. (3.11) can be rewritten as

$$\lfloor A_1 \quad A_2 \quad A_3 \quad A_4 \quad A_5 \rfloor \begin{Bmatrix} K^b \\ C^b \\ K_d \\ K_i \\ U \end{Bmatrix} = B_1 \tag{3.12}$$

Regression matrix on left hand side and known parameters vector on right hand side are the function spin speed. So it is written as

$$A_1(\omega)_{P\times Q} \times (X_1)_{Q\times 1} = B_1(\omega)_{P\times 1} \tag{3.13}$$

Where

$$A_1(\omega)_{P\times Q} = \lfloor A_1 \quad A_2 \quad A_3 \quad A_4 \quad A_5 \rfloor$$

$$(X_1)_{P\times Q} = \{K_{xx1} \quad K_{xy1} \quad \ldots \quad C_{xx1} \quad C_{xy1} \quad \ldots \quad K_{sx1} \quad K_{sy1} \quad \ldots \quad K_{ix1} \quad K_{iy1} \quad \ldots \quad U_1 \quad U_2 \quad \ldots\}^T$$

And U contains the data of residual unbalances at each balancing plane of the system. $A_1(\omega)$ is the regression matrix of the system which contain the values of generated responses corresponding to unknown dynamic parameters. Vector (X_1) contains all dynamic parameters of the system which has to estimate. Subscripts P and Q defines the size of matrix and vectors. The Value of P is dependent on number of master DOFs considered during dynamic condensation and value of Q is dependent on the number of unknown parameters has to identify. Eq. (3.13) becomes

$$[A_1(\omega)]_{8\times 38} \times \{X_1\}_{38\times 1} = \{B_1(\omega)\}_{8\times 1} \tag{3.14}$$

Eq. (3.14) is in the complex form and it can be exploded into real as well as in imaginary parts in such a way that

$$\begin{bmatrix} \left[A_1(\omega)^r{}_{8\times1} \right] \\ \left[A_1(\omega)^i{}_{8\times1} \right] \end{bmatrix}_{16\times38} + \{X_1\}_{38\times1} = \begin{Bmatrix} \{B_1(\omega)\}^r{}_{8\times1} \\ \{B_1(\omega)\}^i{}_{8\times1} \end{Bmatrix}_{16\times1} \qquad 3.15$$

Where r represents real part and i represents imaginary part. Eq. (3.15) can be rewritten as

$$\left[A_2(\omega)\right]_{16\times38} \times \{X_2\}_{38\times1} = \{B_2(\omega)\}_{16\times1} \qquad 3.16$$

Now here we can notice that number of parameters has to estimate (thirty-eight) is more than the number of linear equations (i.e.16) available. So it is an undetermined situation of simultaneous linear equation. The whole unknown parameters must be acquired only when the number of equations will be increased at least equal to or greater than the unknowns. Therefore, here three numbers of independent measurement should be required take so that equations will be 48 and parameters will be easily estimated.

For making the system determinate, there various methods are available such as

(1) Rotate the rotor in the identical direction at different speeds and obtain measurements at each speed.
(2) Rotate the rotor in CW or CCW direction alternatively and obtain measurements.

In this case first method is utilized and rotor is rotated at three different speeds and measurements are done. So Eq. (3.16) becomes

$$\left[A(\omega)\right]_{48\times38} \times \{X\}_{38\times1} = \{B(\omega)\}_{48\times1} \qquad 3.17$$

Where

$$\left[A(\omega)\right]_{48\times38} = \begin{bmatrix} A_2(\omega_1) \\ A_2(\omega_2) \\ A_2(\omega_3) \end{bmatrix} \text{ and } \{B(\omega)\}_{48\times1} = \begin{Bmatrix} B_2(\omega_1) \\ B_2(\omega_2) \\ B_2(\omega_3) \end{Bmatrix}$$

Now Eq. (3.17) becomes determinate and parameters can be estimated as follows

$$\{X\} = \left[\left[A(\omega)\right]^T \left[A(\omega)\right]\right]^{-1} \left\{\left[A(\omega)\right]^T \{B(\omega)\}\right\} \qquad 3.18$$

Elements of $A(\omega)$ and $B(\omega)$ are explained in detail in appendix.

A detail procedure for estimating the dynamic parameters and residual unbalances in system using identification algorithm is explained in flow chart shown Figure 3.1

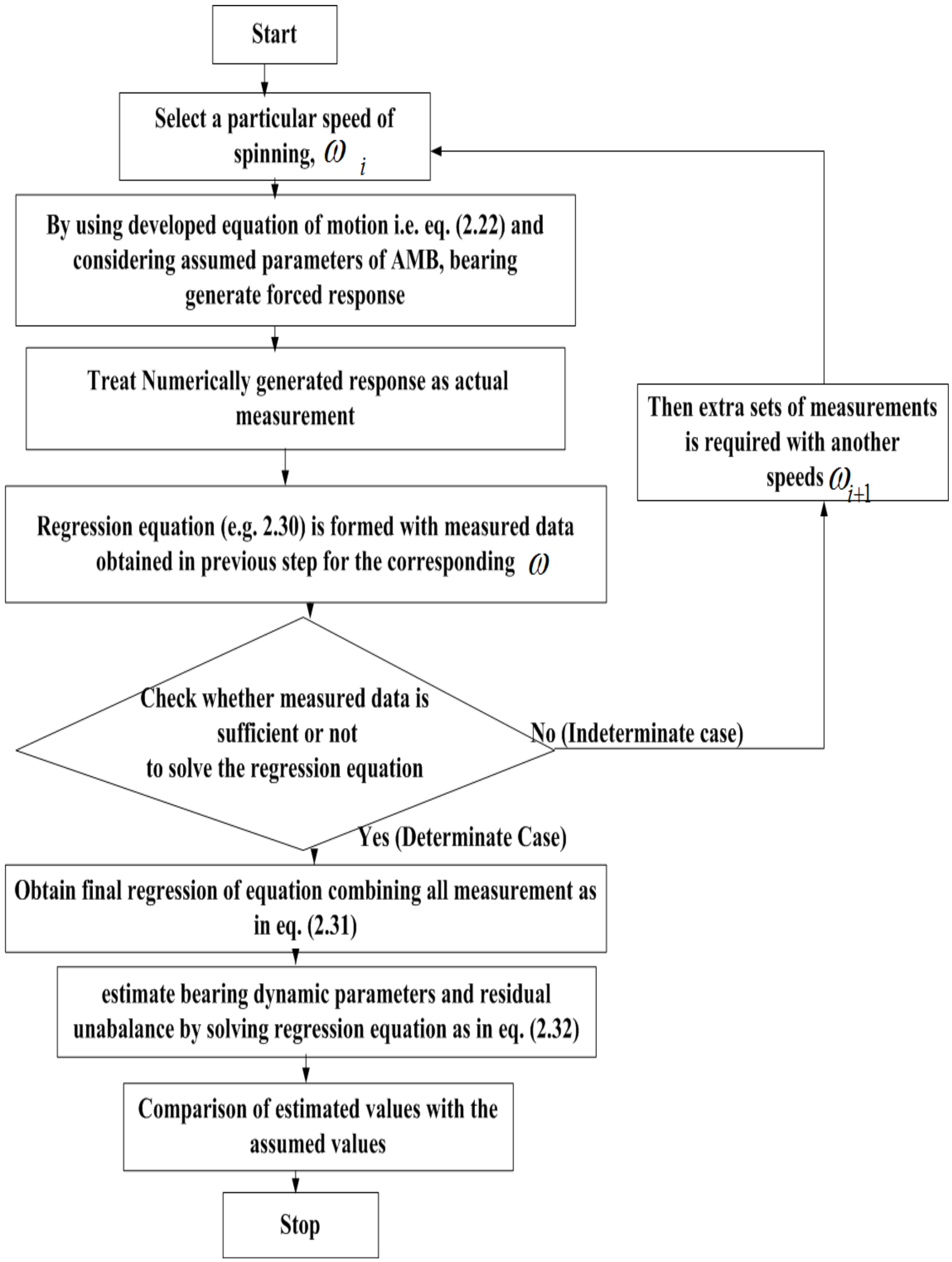


Figure 3.1 Flowchart of Identification Algorithm

Chapter 4

Numerical Simulation

The proposed formulation is given for a simple 4-DOF flexible rotor system, that is shown in Figure 2.2, supporting with two flexible bearings at both end and two AMBs are used to generate controlling forces as explained before is tested with the help of a direct numerical simulation technique using MATLAB. Physical properties of the flexible rotor which are considered for numerical analysis are explained in Table 4.1. Physical characteristics and dimensions of rotor are considered based on the conceivable laboratory rotor test kit and it is reported by Shravnkumar and Tiwari (2013).

Table 4.1 Physical Characteristics of rotor

S.N.	Parameters	
1.	Mass Density, ρ	7850 Kg/m^3
2.	Young's Modulus, E	210 GPa
3.	Modulus of Rigidity, G	76 GPa
4.	Diameter of rotor, d and rotor total length L	10 mm and 1 m
	Balancing mass (m) and eccentricities (ℓ)	
	Disc (1)	20gm, 5mm
	Disc (7)	20gm, -5mm
	Disc (2)	10gm, -3mm
	Disc (3)	5gm, -1mm
	Disc(4)	0,0
	Disc (6)	10gm, 3mm
	Disc (5)	5gm, -1mm

4.1 Discussion of Simulink model

Simulink model developed with the use of Eq. (2.10) for numerical simulation for obtaining displacement and current responses in time domain is shown in Figure 4.1. SIMULINK™ block of 'clock' is used to maintain time of simulation. Triangular block contains gain parameters, which make multiplication of the input coming. System operational parameters like stiffness, damping, mass as well as omega is kept in such block. SIMULINK™ block of Trigonometric function like 'sine' and 'cosine' are used to perform trigonometric operations on input signals. Some constants like phase, dynamic matrix, etc. are placed in constant blocks. Algebraic summation of the input signal has been done according to sequences as '+' and '-' signs appear by using sum block. Calculus operations are performed by using integrator and derivative block.

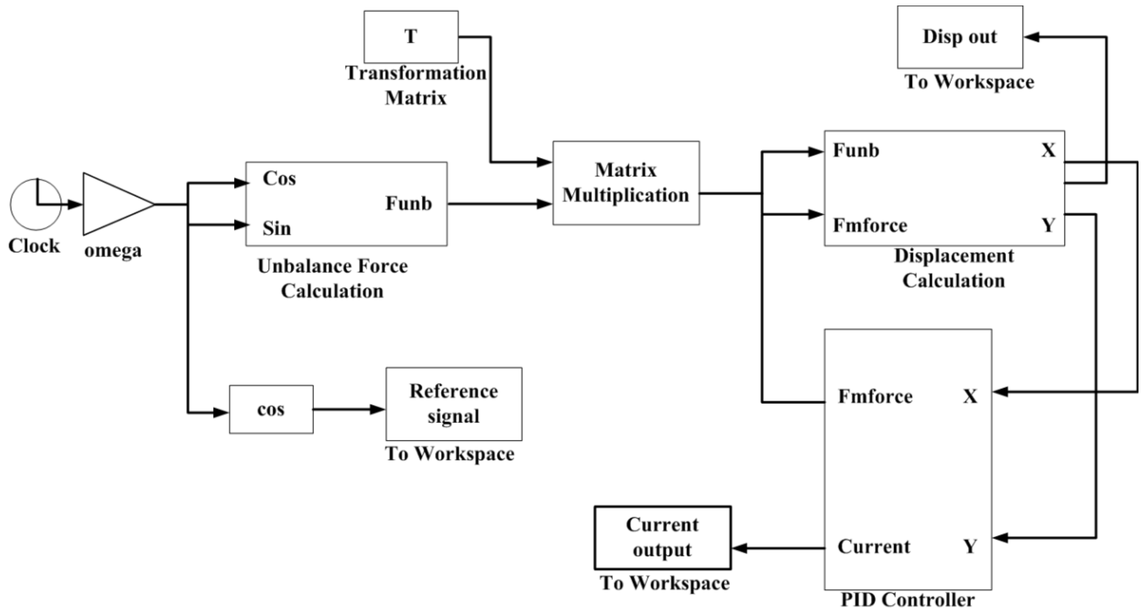
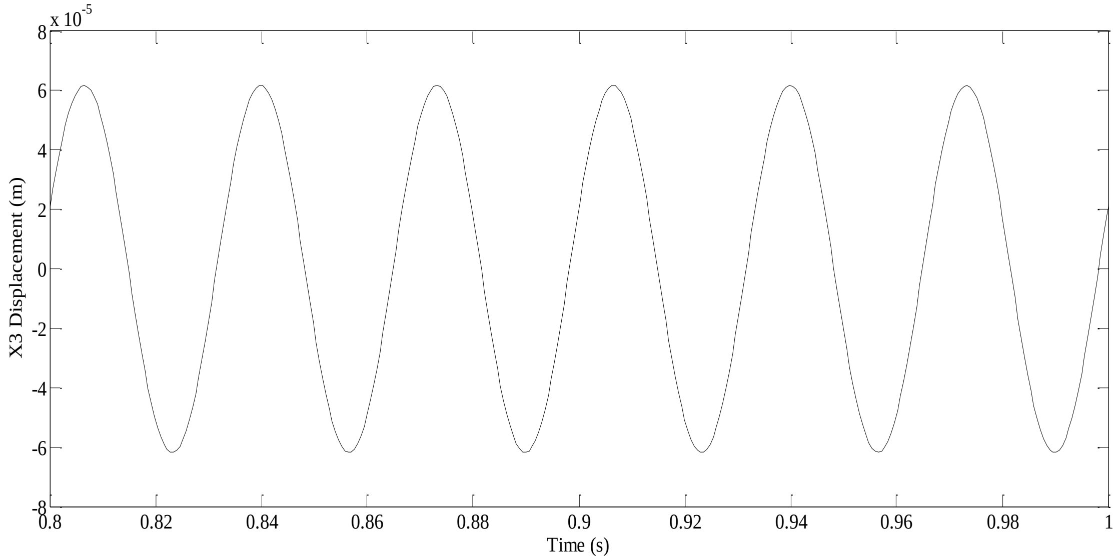


Figure 4.1 Simulink Model

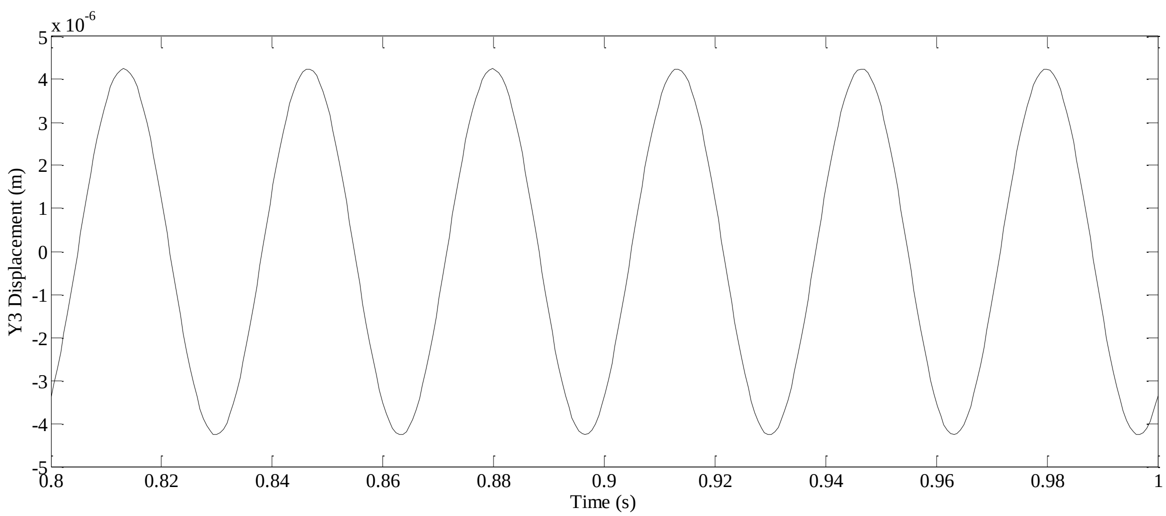
A Standard PID controller is employed in the model. Gain parameters of PID controller are considered for the simulation based on a study done by Bordoloi and Tiwari (2013) on the optimization of performance of gain parameters. The PID controller has properties: K_p = 4200 Amp/m, K_I =2000 Amp/m-s, K_D = 10.0 Amp-s/m. Here with the use of fourth order Runge-Kutta solver, forced response is generated. A forced response contains all the linear displacement at all nodal locations of the rotor as well as current at AMB locations. Displacements obtained at various nodes are passed through PID Controller. The PID controller calculates current and then controlling forces will be generated by active magnetic bearing. The Present Simulink model is run for 5s with step size of 0.00002s and response data is obtained. Out of that, response data for 2-5s (3s time duration) is considered for analysis.

4.2 Response in time and frequency domain

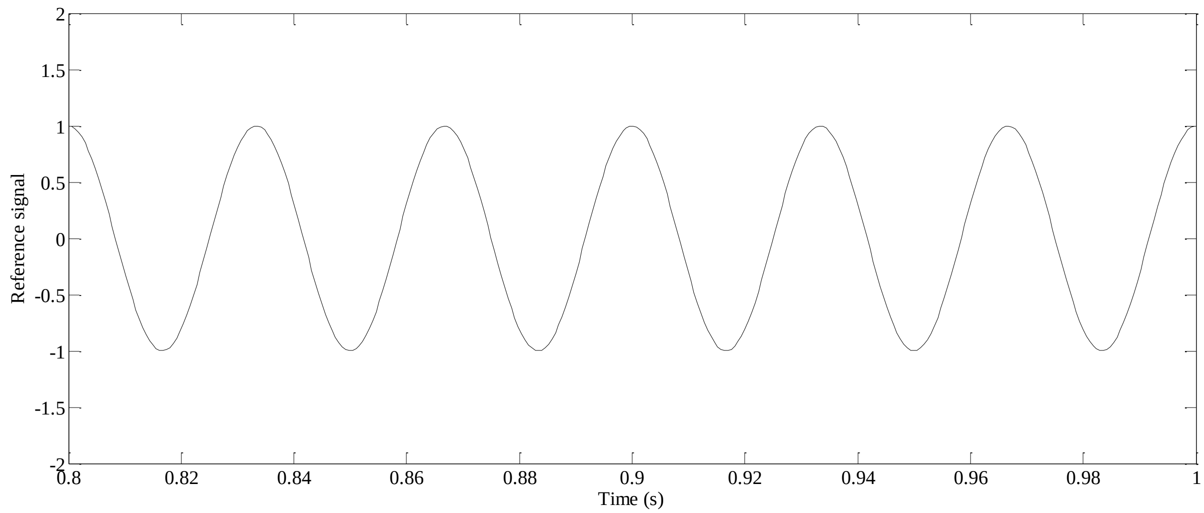
A typical forced response is generated with numerical simulation in time domain as well as in frequency domain at three different spin speeds i.e. 188.49rad/sec, 376.9911rad/sec and 565.4867rad/sec to overcome the difficulty of underdetermined and it is shown in following figures. A typical numerically simulated displacement characteristic in Time Domain with reference signal is shown in Figure 4.2



(a) X-displacement



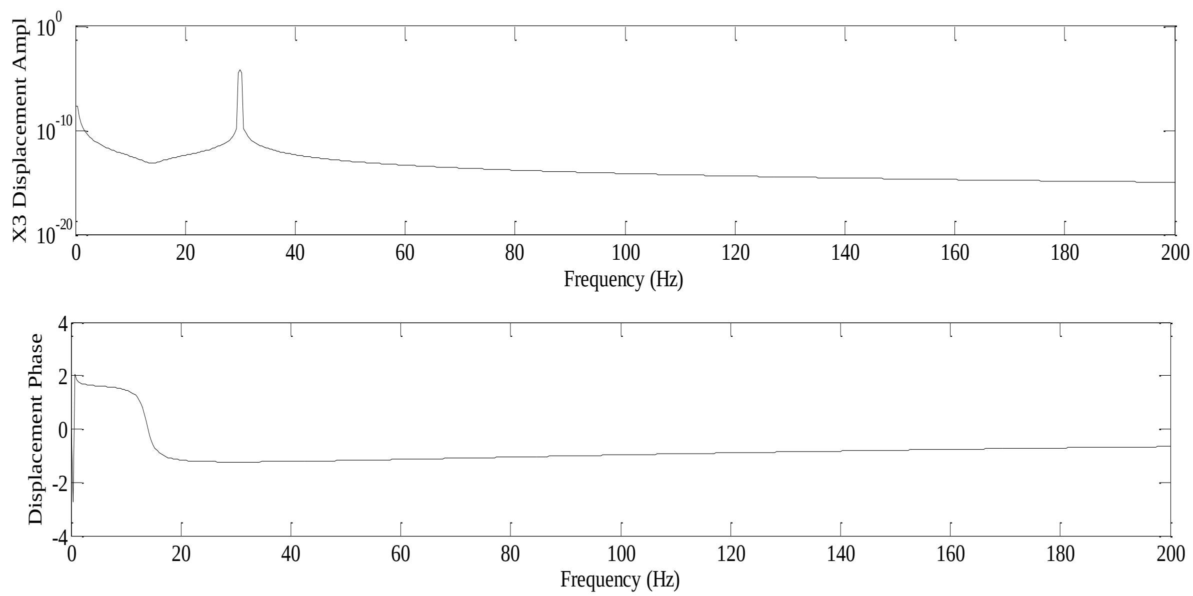
(b) Y-displacement



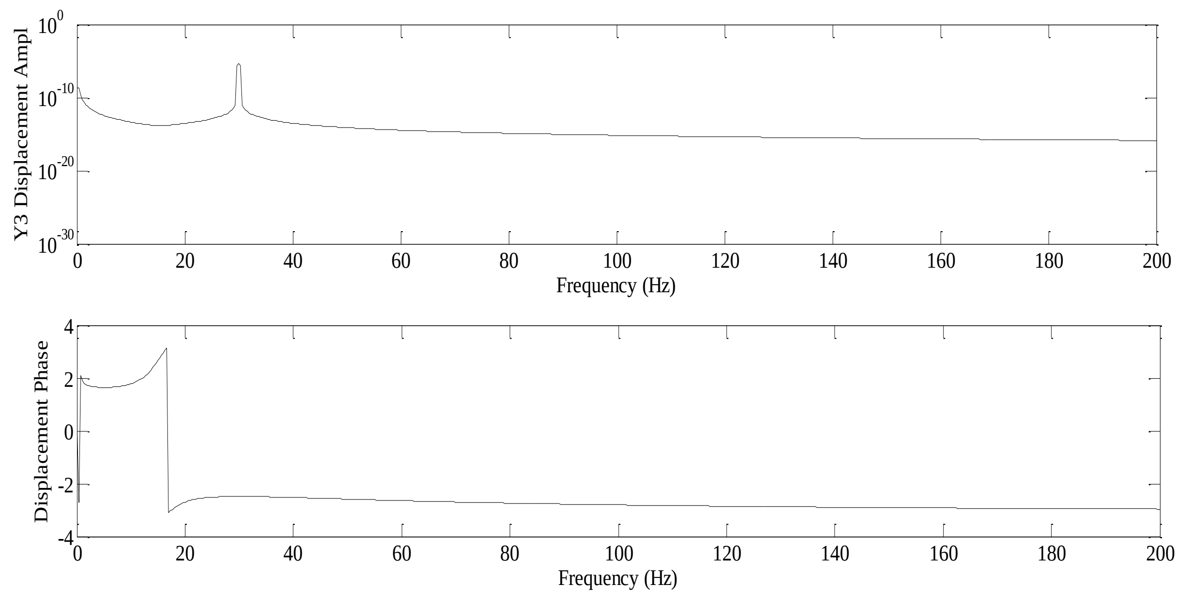
(c) Reference Signal

Figure 4.2 Displacement characteristic in Time Domain (a) X3 displacement, (b) Y3 displacement and (c) reference signal

Obtained time domain is afterward transformed into frequency domain as shown in Figure 4.3 by using fast Fourier transformation (FFT).



(a) Amplitude of X3 Displacement

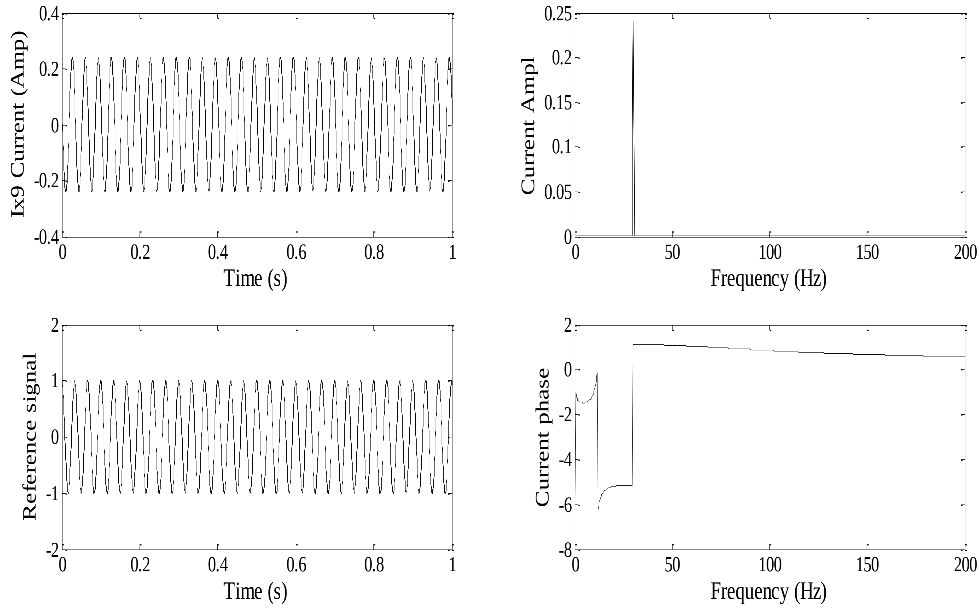


(b) Amplitude of Y3 Displacement

Figure 4.3Displacement characteristic in Frequency Domain

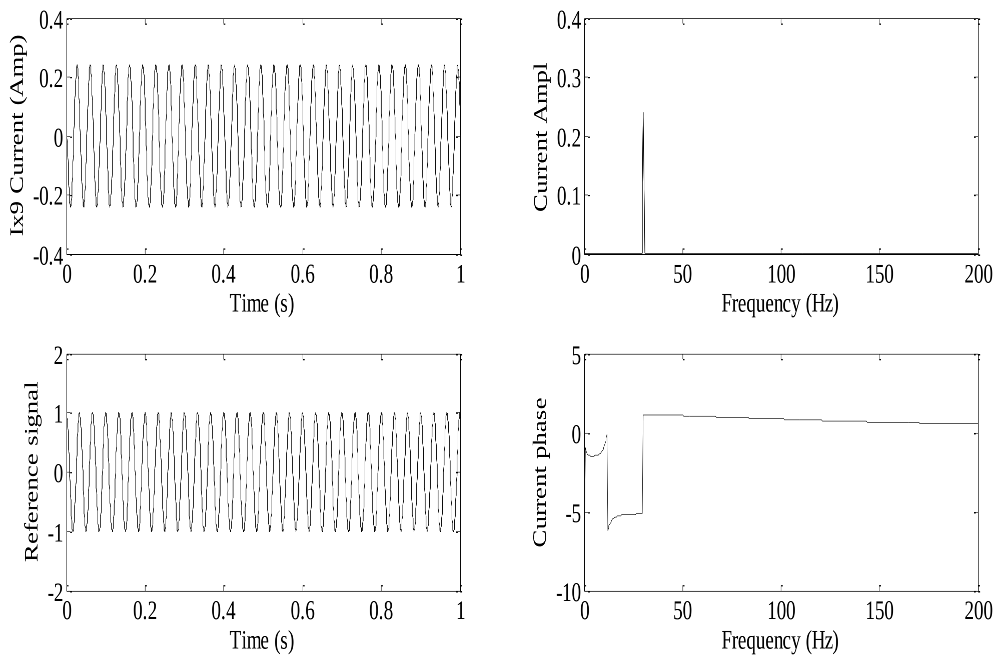
(a) Amplitude of X3 Displacement and (b) Amplitude of Y3 Displacement

Forced responses of displacement are given to the PID controller. Afterward, it converts the input displacement signal into the current signal which is then given to the electro-magnets and levitation force will be generated according to the position of the rotor. This controlling force suppresses the vibration produced in the system. The information about current data based on displacement data at locations of AMB are obtained in both domains and it is shown in Figure 4.4



(a)

Ix9 Current



(b) Iy9 Current

Figure 4.4 Characteristic of current in time and Frequency Domain (a) Characteristics of Ix9 current and (b) Characteristics of Iy9 current

With the use of this response, all dynamic parameters of the system are estimated and these are written in detail in following tables. Table 3.2 and 3.3 shows the stiffness and damping parameters of conventional bearing respectively.

Table 4.2 Conventional Bearing assumed and obtained stiffness parameters comparison

Dynamic Parameters used for simulation (N/m)	Assumed values	Estimated parameters		
		Without Noise	1% Noise	5% Noise
K_{xx1}	2.5×10^5	2.3152×10^5	2.2552×10^5	2.1552×10^5
K_{xy1}	1.20×10^5	1.2070×10^5	1.1870×10^5	1.1070×10^5
K_{yx1}	1.35×10^5	1.3155×10^5	1.2955×10^5	1.1555×10^5
K_{yy1}	2.75×10^5	2.1473×10^5	2.1073×10^5	2.0173×10^5
K_{xx2}	2.14×10^5	2.0564×10^5	2.0064×10^5	1.9506×10^5
K_{xy2}	1.21×10^5	1.1864×10^5	1.1464×10^5	1.0464×10^5
K_{yx2}	1.29×10^5	1.1634×10^5	1.1334×10^5	1.0334×10^5
K_{yy2}	2.56×10^5	1.9310×10^5	1.8510×10^5	2.9010×10^5

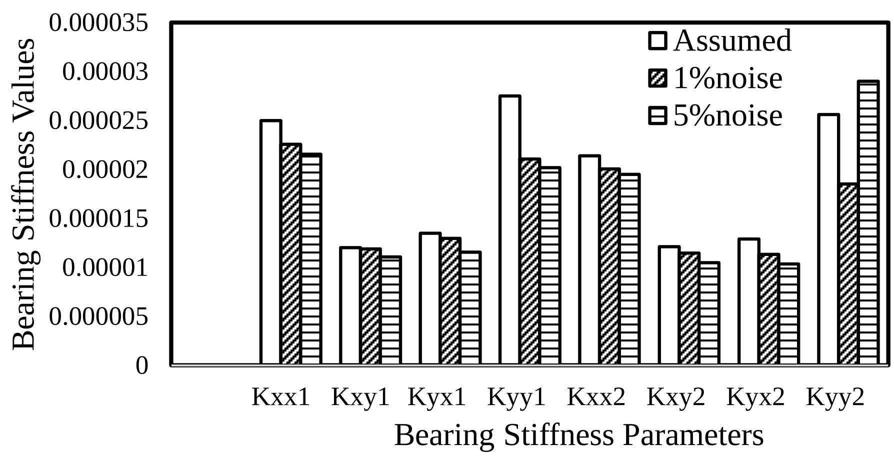


Figure 4.5 Comparison between assumed and identified Stiffness parameter of Bearing

Table 4.3 Conventional Bearing assumed and obtained stiffness parameters comparison

Dynamic Parameters used for simulation (Ns/m)	Assumed values	Estimated parameters		
		Without Noise	1% Noise	5% Noise
C_{xx1}	300	235.7221	225.7221	200.7221
C_{xy1}	20	24.8314	27.8314	10.8314
C_{yx1}	50	64.8700	65.8700	75.8700
C_{yy1}	399	327.3175	325.3175	295.3175
C_{xx2}	335	337.6400	333.6400	370.6400
C_{xy2}	64	66.8908	68.8908	70.8908
C_{yx2}	55	78.8820	80.8820	45.8820
C_{yy2}	295	237.6122	230.6122	260.6122

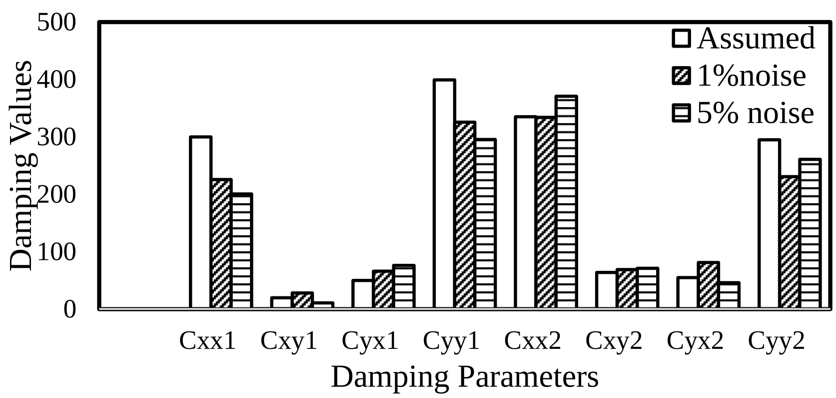


Figure 4.6 Comparison between assumed and identified damping parameter of Bearing

Table 3.4 shows the AMB dynamic parameters, i.e. displacement stiffness and current stiffness. First four rows of the table 3.4 shows the displacement stiffness values and remaining row displays the current stiffness values of AMB. First and second column of tables contains the parameters, whichare assumed during simulation for generation of responses. Third column of tables shows the estimated parameters without adding error in responses.

Table 4.4 AMB assumed and obtained stiffness parameters comparison

Dynamic Parameters used for simulation	Assumed values	Estimated parameters		
		Without Noise	1% Noise	5% Noise
K_{sx1} (N/m)	1.05×10^5	1.0728×10^5	1.1128×10^5	1.3028×10^5
K_{sy1} (N/m)	1.25×10^5	1.1716×10^5	1.1516×10^5	1.3516×10^5
K_{sx2} (N/m)	1.30×10^5	1.2513×10^5	1.2013×10^5	1.1013×10^5
K_{sy2} (N/m)	1.50×10^5	1.3213×10^5	1.2513×10^5	1.0513×10^5
K_{ix1} (N/A)	85	80.1604	78.1604	98.1604
K_{iy1} (N/A)	68	69.7729	70.7729	72.7729
K_{ix2} (N/A)	80	87.5034	89.5034	95.5034
K_{iy2} (N/A)	73	57.4454	54.4454	45.4454

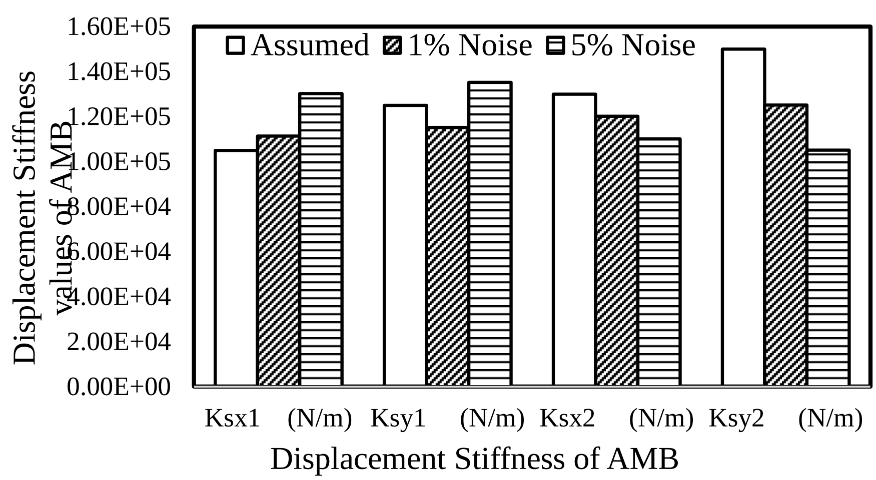


Figure 4.7 Comparison between assumed and identified Displacement-stiffness parameter of AMB

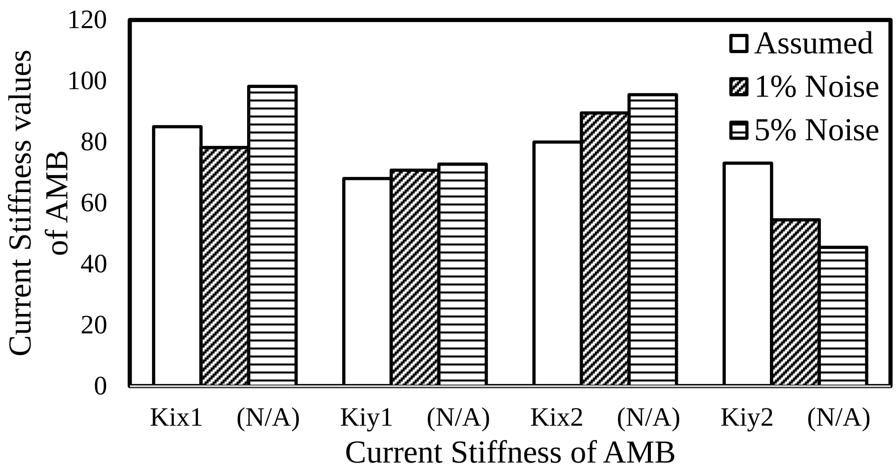


Figure 4.8 Comparison between assumed and identified Current-stiffness parameter of AMB

Table 3.5 indicates the assumed residual unbalance during obtaining forced response and estimated unbalances magnitude using the proposed algorithm. It shows that parameters obtained without noise are close to the assumed values. For checking the robustness of the proposed methodology, algorithm is tested for the signals with some percent of error in it. Then it is notified that as the percentage of noise increases in forced responses, the values of estimated parameters will starts deviating from assumed values. For 1% error, estimated values are somewhat nearer to the assumed one. But for 5% noise, estimated values varies substantially.

Table 4.5 Assumed and estimated magnitude of unbalances

Dynamic Parameters used for simulation $(kg-m)$	Assumed values	Estimated parameters		
		Without Noise	With 1% Noise	With 5% Noise
U_{x1}^{r}	8.6602×10^{-5}	2.6378×10^{-5}	2.1378×10^{-5}	4.1378×10^{-5}
U_{x1}^{i}	5.00×10^{-5}	-6.4270×10^{-5}	-5.4270×10^{-5}	-7.4270×10^{-5}
U_{x2}^{r}	-5.209×10^{-6}	-1.572×10^{-6}	-5.725×10^{-6}	-6.725×10^{-6}
U_{x2}^{i}	-2.954×10^{-5}	5.2292×10^{-5}	6.2292×10^{-5}	7.2292×10^{-5}
U_{x3}^{r}	3.8302×10^{-6}	1.3294×10^{-5}	1.0294×10^{-6}	3.0294×10^{-6}
U_{x3}^{i}	-3.213×10^{-6}	-0.000055237	-4.5237×10^{-5}	-2.5237×10^{-5}
U_{x4}^{r}	0	-1.1264×10^{-4}	-2.1264×10^{-4}	-5.1264×10^{-4}
U_{x4}^{i}	0	5.3961×10^{-4}	6.3961×10^{-4}	6.0961×10^{-4}
U_{x5}^{r}	-8.6×10^{-7}	8.642×10^{-6}	9.642×10^{-6}	8.642×10^{-6}
U_{x5}^{i}	-4.92×10^{-6}	-4.6538×10^{-5}	-6.6538×10^{-5}	-7.6538×10^{-5}
U_{x6}^{r}	1.846×10^{-5}	-4.976×10^{-5}	-3.976×10^{-5}	-4.976×10^{-5}
U_{x6}^{i}	-2.364×10^{-5}	1.945218×10^{-5}	2.245218×10^{-4}	2.23218×10^{-4}
U_{x7}^{r}	9.6596×10^{-5}	1.056554×10^{-5}	1.156554×10^{-5}	1.056554×10^{-5}
U_{x7}^{i}	9.6596×10^{-5}	2.854052×10^{-5}	3.854052×10^{-5}	4.854052×10^{-5}

Percentage errors are also calculated between the assumed values and estimated values with 1% noise and 5% noise and it is shown in Figure 4.9. Square points indicate the percentage error with 1% noise and circle indicates error percentage with 5% noise. From the graph itself,it can be notifying that error with 1% noise signal is pretty less, but as percentage of noise growing, value of estimation deviates more so that percentage of error is somewhat significant.

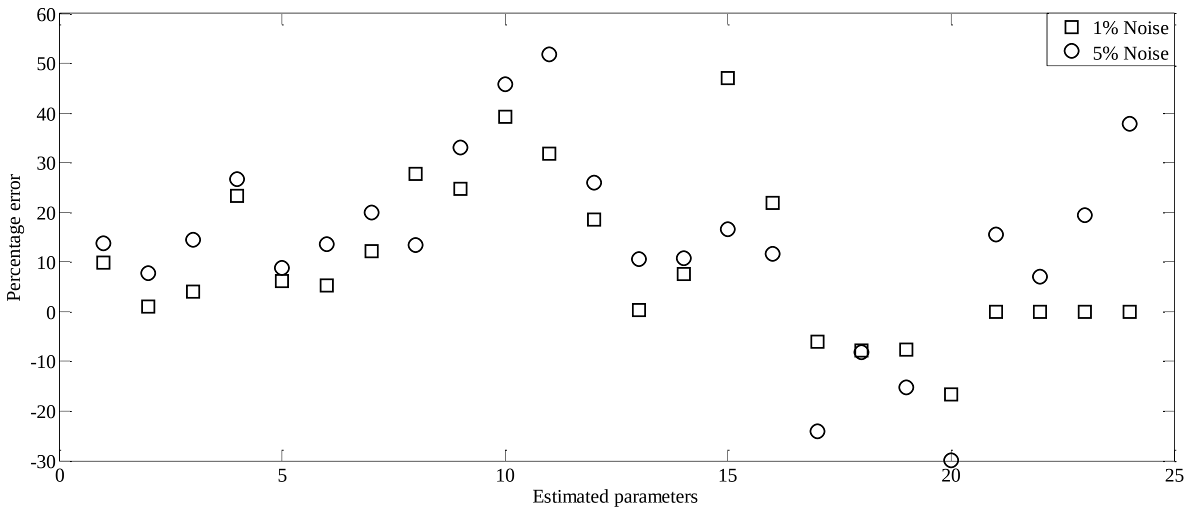


Figure 4.9 Error comparison of estimated values at different noise signals

Practically because of measurement restriction, it is not feasible to acquire the precise values of model parameters always. Therefore, for testing the proposed algorithm against the modeling errors (bias error), errors in the rangeof 1%, 2% and 5% are intentionally introduced in numerical modelparameters such as the mass, damping, stiffness.Initially with the correct parameters of modelnumerical response is generatedand then the error is introduced inthe model parameters that are then used for estimation inidentification algorithm. Therefore, we have a correct response butthe model used for estimation, is having error. Now with this correctlygenerated responses and model parameters with an error, estimates are obtained which are compared with assumed values. It is seen that the identification algorithm is very much robust against modeling errors.

Chapter 5

Conclusion

In the present work, an identification algorithm is proposed for simultaneous estimation of equivalent magnitude of continuous unbalance in flexible rotor for the case where rotor is supported with two conventional bearings at its end and two AMB are used to suppress the unbalance force. Forced response, i.e. displacements at all nodes of the rotor as well as current at AMB location, are generated by the use of MATLAB SIMULINK. The generated response in the time domain is translated into frequency domain because the present algorithm requires a forced response in frequency form. Considering this forced response as actual measurement and using it into developed algorithm, dynamic parameters of conventional bearing and AMB are estimated as well as the magnitude of unbalance forces is estimated. Parameters are estimated against noisy signal also and robustness of it is analyzed. Theflexible rotor system is modeled and equation of motion for the system is obtained by using the Finite element method. In the current work, Effect of the foundation of the system is not considered.

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APPENDIX A: Matrices of Equation of Motion

A.1. Elemental mass matrix

$$[M]^e = \begin{bmatrix} 156 & 0 & 0 & 22l & 54 & 0 & 0 & -13l \\ & 156 & -22l & 0 & 0 & 54 & 13l & 0 \\ & & 4l^2 & 0 & 0 & -13l & -3l^2 & 0 \\ & & & 4l^2 & 13l & 0 & 0 & -3l^2 \\ & & & & 156 & 0 & 0 & -22l \\ & & & & & 156 & 22l & 0 \\ & \text{Sym} & & & & & 4l^2 & 0 \\ & & & & & & & 4l^2 \end{bmatrix} \qquad \text{(A.1)}$$

Displacement vector $\{\eta\}^{ne} = \left\{x_i, \varphi_{y_i}, y_i, \varphi_{x_i}, x_{i+1}, \varphi_{y_{i+1}}, y_{i+1}, \varphi_{x_{i+1}}\right\}^T$, where sub–script i is element number

A.2. <u>Elemental stiffness matrix</u>

$$\left[K\right]^e = \begin{bmatrix} 12 & 0 & 0 & 6l & -12 & 0 & 0 & 6l \\ & 12 & -6l & 0 & 0 & -12 & 6l & 0 \\ & & 4l^2 & 0 & 0 & 6l & 2l^2 & 0 \\ & & & 4l^2 & -6l & 0 & 0 & 2l^2 \\ & & & & 12 & 0 & 0 & -6l \\ & & & & & 12 & 6l & 0 \\ & \text{Sym} & & & & & 4l^2 & 0 \\ & & & & & & & 4l^2 \end{bmatrix} \qquad \text{(A.2)}$$

A.3. Stiffness matrix of the conventional bearing

$$Kc = \begin{bmatrix} k_{xx1} & k_{xy1} & 0 & 0 \\ k_{yx1} & k_{yy1} & 0 & 0 \\ 0 & 0 & k_{xx4} & k_{xy4} \\ 0 & 0 & k_{yx4} & k_{yy4} \end{bmatrix} \tag{A.3}$$

A.4.Damping matrix of the conventional bearing

$$Cc = \begin{bmatrix} c_{xx1} & c_{xy1} & 0 & 0 \\ c_{yx1} & c_{yy1} & 0 & 0 \\ 0 & 0 & c_{xx4} & c_{xy4} \\ 0 & 0 & c_{yx4} & c_{yy4} \end{bmatrix} \tag{A.4}$$